Algebraic dynamic multilevel (ADM) method for compositional multi-phase flow simulation


*College of Aerospace and Civil Engineering, Harbin Engineering University, Harbin 150001 China
E-mail: sunxiaoyu520634@163.com, guxuan@hrbeu.edu.cn, wangbinsheng@hrbeu.edu.cn
E-mail:hezheng@hrbeu.edu.cn,sunxiaoyu520634@163.com,wangbinsheng@hrbeu.edu.cn

ABSTRACT

We describe a sequential fully implicit (SFI) multi-scale finite volume (MSFV) algorithm for nonlinear multi-phase flow and transport in heterogeneous porous media. The method extends the recently developed multiscale approach, which is based on an IMPES (Implicit Pressure, Explicit Saturation) scheme. That previous method was tested extensively and with a series of difficult test cases, where it was clearly demonstrated that the multiscale results are in excellent agreement with reference fine-scale solutions and that the computational efficiency of the MSFV algorithm is much higher than that of standard reservoir simulators. However, the level of detail and range of property variability included in reservoir characterization models continues to grow. For such models, the explicit treatment of the transport problem (i.e. saturation equations) in the IMPES-based multiscale method imposes severe restrictions on the time step size, and that can become the major computational bottleneck. Here we show how this problem is resolved with our sequential fully implicit (SFI) MSFV algorithm. Simulations of large (million cells) and highly heterogeneous problems show that the results obtained with the implicit multi-scale method are in excellent agreement with reference fine-scale solutions. Moreover, we demonstrate the robustness of the coupling scheme for nonlinear flow and transport, and we show that the MSFV algorithm offers great gains in computational efficiency compared to standard reservoir simulation methods.

ARTICLE DETAILS

1. INTRODUCTION

The level of detail in reservoir characterization models continues to grow. This is driven by the need to improve the predictive capacity of dynamic simulations of the complex reservoir displacement processes of practical interest. These highly detailed heterogeneous reservoir descriptions exceed the computational capability of existing reservoir simulators. This resolution gap is usually tackled by upscaling the fine-scale description to sizes that can be treated by the numerical simulator. In upscaling, the original model is coarsened using a computationally inexpensive process. In purely local flow-based upscaling methods [1,2], the process is usually based on single-phase flow. The simulation study of the multi-phase reservoir displacement process of interest is then performed using the coarsened (upscaled) model, which is derived using a simple flow process. These upscaling methods have proved quite successful.

Jenny et al[3]. developed a multi-scale formulation for the flow problem (pressure and total-velocity) that fits nicely into a finite-volume framework. In that multi-scale finite-volume (MSFV) method, one set of basis functions is used to obtain effective coarse-scale transmissibilities, and a second set of basis functions is devised to ensure that the reconstructed fine-scale total-velocity field is conservative.

II. IMPLICIT MULTI-SCALE FINITE-VOLUME METHOD

The implicit MSFV approach for multi-phase flow and transport is based on the multi-scale method of Jenny et al[3,4]. First we briefly outline the MSFV method for dealing with the flow problem, then we describe our Sequential Fully Implicit (SFI) multiscale finite-volume algorithm for coupled nonlinear flow and transport. We use a sequential solution strategy in our MSFV method, where flow (pressure and total velocity) and transport (saturation) are treated separately and differently. The algorithms are specialized to the specific characteristics of the governing equations. In our Sequential Fully Implicit (SFI) MSFV algorithm, each time step consists of an outer loop to solve the coupled problems of flow and transport, and an inner (nonlinear) Newton loop to solve the implicit transport problem given the updated flow field.

A flow diagram of the outer loop for one time step is shown in Fig. 2. The superscripts $n$ and $v$ denote the old time and iteration levels, respectively. Saturation is represented by $S$, the total velocity field by $u$, the computation of the pressure field by the operator $P$, and the computation of the transport problem (i.e. saturation field) by $T$. We solve for, and we update the total velocity field. Now, given, a second, or inner, Newton loop is used to solve the saturation equation for the current time step implicitly (operator $T$ in Fig.1). The semi-discrete form of the saturation equation is given by

$$ S = S $$

with

**Fig.1 Flow diagram of the sequential fully implicit MSFV scheme.**

To study the accuracy and efficiency of the SFI-MSFV algorithm, 2D and 3D test cases with uniformly spaced orthogonal $60 \times 220$ and $60 \times 220 \times 85$ grids were used. The 3D grid and permeability field are obtained from the SPE 10 test case [5], which is known to be an
extremely difficult problem for reservoir simulators. While the 3D test case was used for computational efficiency assessment, 2D test cases, which consist of the top and bottom layers of the SPE 10 model, are used to demonstrate the accuracy of the SFI-MSFV method. Fig.2 shows the permeability field with a water injector in the center and four producers wells in the corners. These well locations are used for all the following studies. The reservoir is initially fully saturated with oil. We take \( M = \frac{\mu_o}{\mu_w} = 10 \), and use quadratic relative permeability curves. These choices lead to a nonlinear coupling of flow and transport, and that is an essential feature of reservoir simulation problems of practical interest.

Fig.2 3D test case: permeability field of the SPE 10 problem (darker means lower permeability).

III. NUMERICAL RESULTS

A.3.1 2D simulations of the top and bottom layers

To do the accuracy assessment in 2D, i.e. with the top and the bottom layers of the 3D model. These two layers, for which the permeability fields are shown in Fig.3, are representative for the two characteristically different regions of the full model. In Figs.4 and 5, the computed saturation fields are shown after 0.093 PVI for the bottom and top layers, respectively. It can be observed for both layers that the agreement is excellent, and that the multi-scale method is not sensitive to the choice (size) of the coarse grid. More quantitative comparisons are shown in Fig.6 where the fine-scale and multi-scale oil cut (fraction of fluid produced that is oil) and cumulative recovery curves are plotted. Considering the difficulty of these test problems and the fact that two independently implemented simulators are used for the comparisons, this agreement is remarkable.

Fig.3 Permeability fields of the bottom (left) and top (right) layers (darker means lower permeability).

Fig.4 The saturation fields (dark is water) from three different simulations of the bottom layer of SPE 10.

B. 3D simulations

While 2D studies are appropriate to study the accuracy of the sequential fully implicit MSFV method, 3D computations for highly detailed models with strong permeability heterogeneity are required for a meaningful assessment of the computational efficiency. A coarse 10×22×17 grid (shown in Fig.7) was used and 0.5 pore volumes were injected.

Fig.7 3D test case

Fig.8 Oil cut and oil recovery.

Fig.8 shows the oil cut and recovery curves obtained with multi-scale simulations using 50 and 200 time steps. The close agreement between the results indicates that the accuracy of the computed flow response is not strongly sensitive to the time step size used. The sequential fully implicit coupling scheme allows for specializing the solution methods to the flow and transport components, which seems to improve the robustness of the outer and inner nonlinear loops with respect to the time step size. This is an area of ongoing research. Finally, for a number of highly heterogeneous systems, we have observed that the cost of MSFV simulation scales almost linearly with problem size, and since the dual and primal basis functions can be computed independently (and adaptively), the method is ideally suited for parallel computations and large-scale problems of practical interest.
IV. CONCLUSIONS

An adaptive sequential fully implicit multi-scale finite-volume (SFI-MSFV) method for 3D nonlinear multiphase flow and transport in heterogeneous porous media was developed and tested for various difficult test cases. The method has proved to be very efficient, and the results are in excellent agreement with reference fine-scale solutions. While the discussion here was limited to incompressible nonlinear two-phase flow, the method is equally applicable to three-phase flow. In that case, one must solve two saturation equations implicitly in the inner loop of the coupling scheme.

ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China (No. 11602066) and the National Science Foundation of Heilongjiang Province of China (QC2015058 and 42400621-1-15047), the Fundamental Research Funds for the Central Universities.

References


